Error Analysis of Noisy 3-qubit Modified Grover's Algorithms

Tyler King

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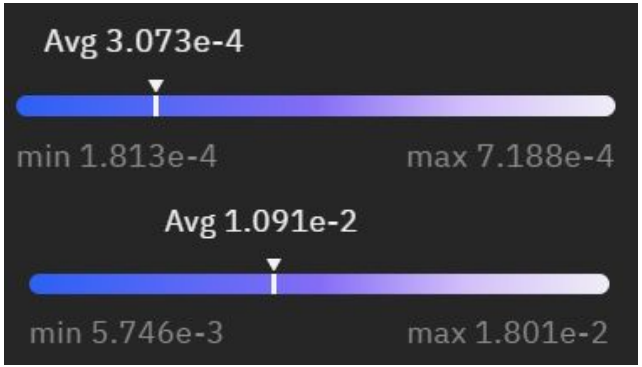
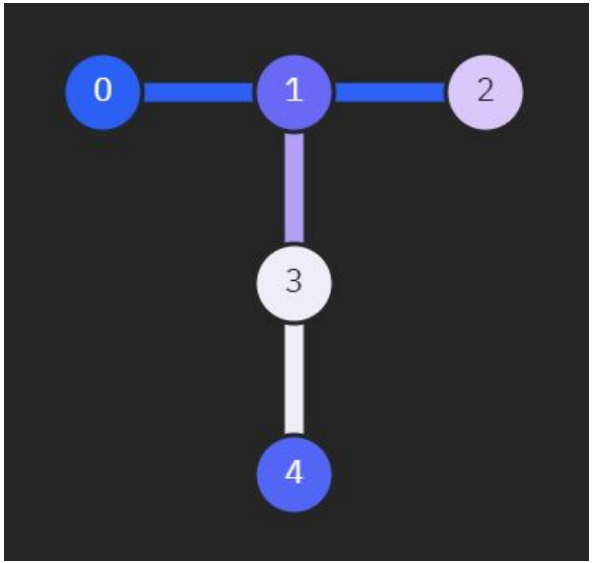
Current Issues

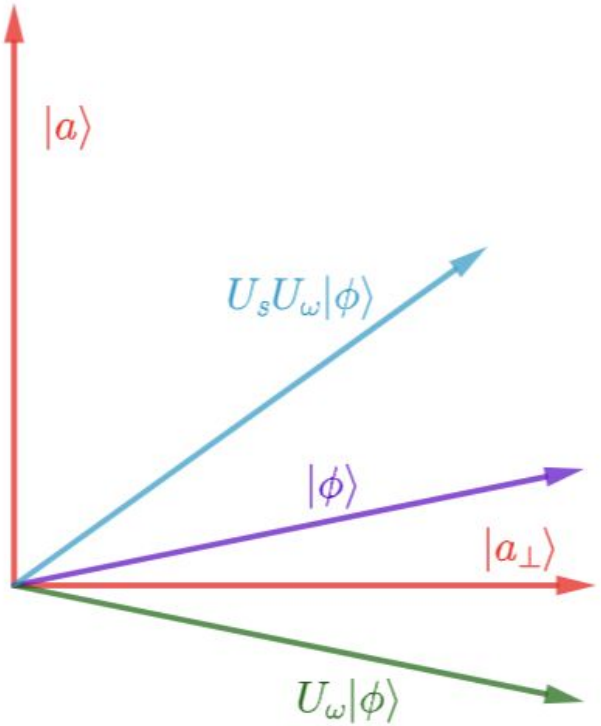
- High **error rates** on quantum hardware
  - Caused by single- and multi-qubit error propagation
- Lack of qubit connections





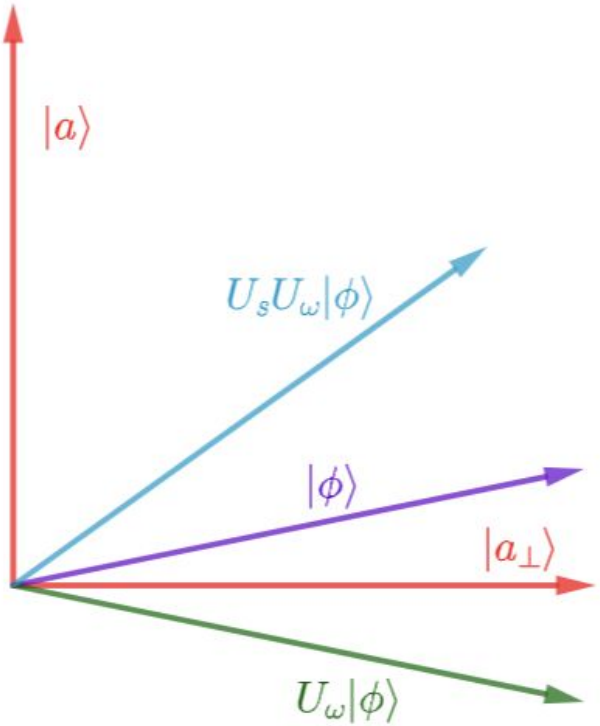
Grover's Algorithm

- Search algorithm
- $O\left(\sqrt{N}\right)$ runtime
  - Quadratic speedup

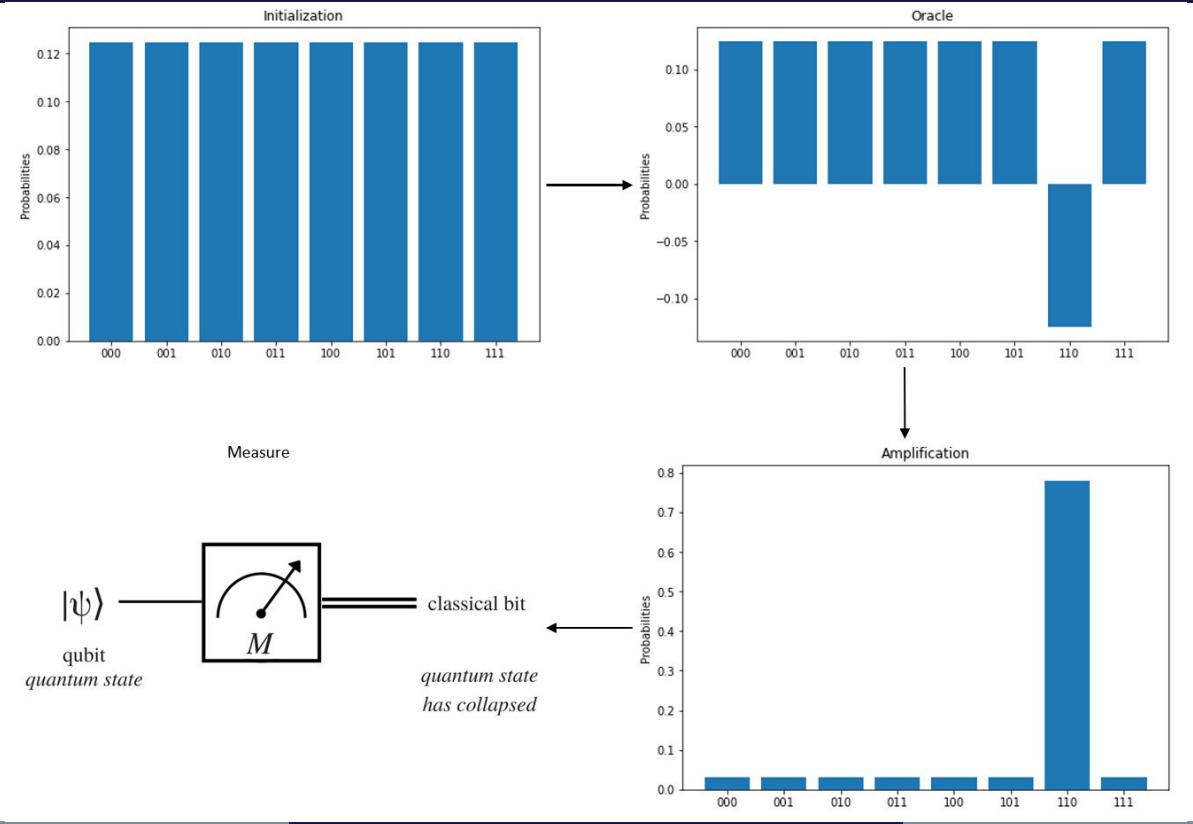


Grover's Algorithm (continued)

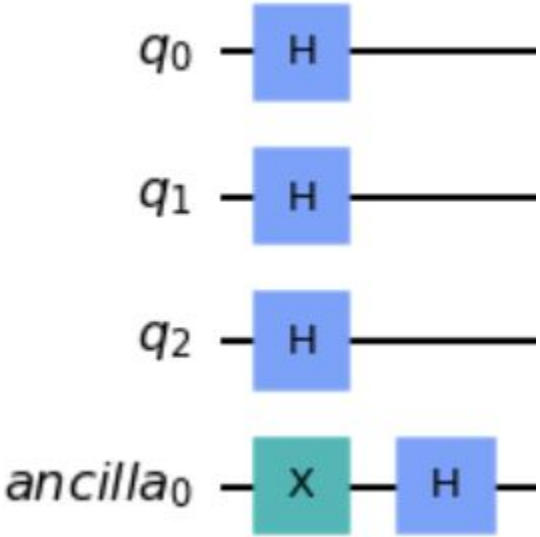
- Initialization: create superposition state
- Oracle stage: flip target state
- Amplification stage: flip across mean
- Measure stage: check qubit state



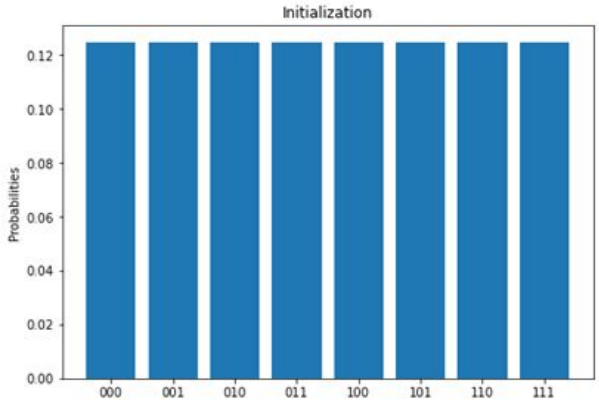
Grover's Algorithm: 4 stages



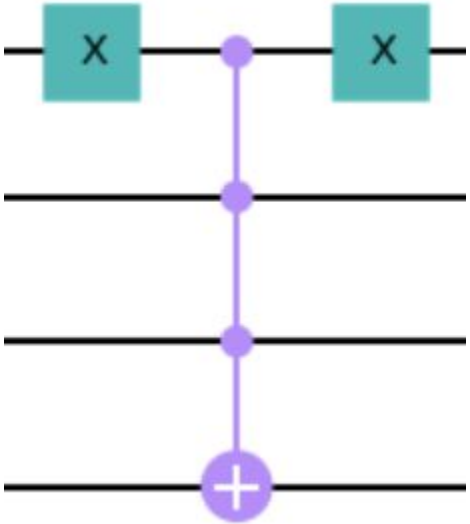
Initialization Stage

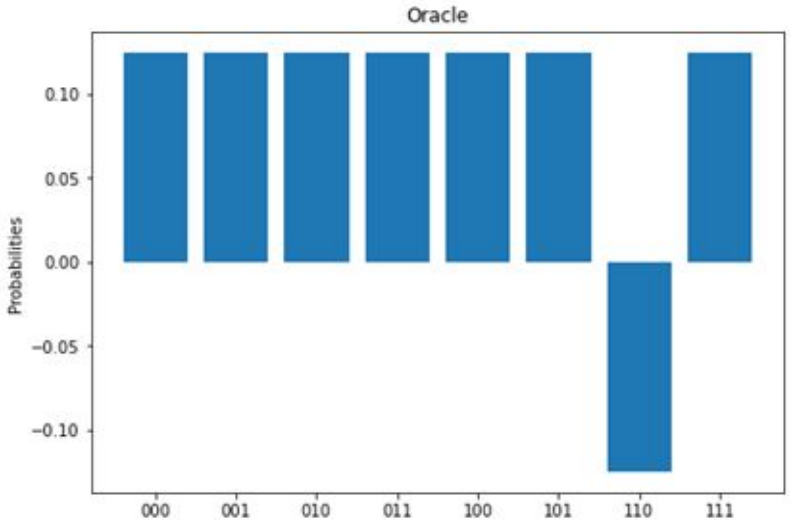


$$H^{\otimes n}$$



Oracle Stage



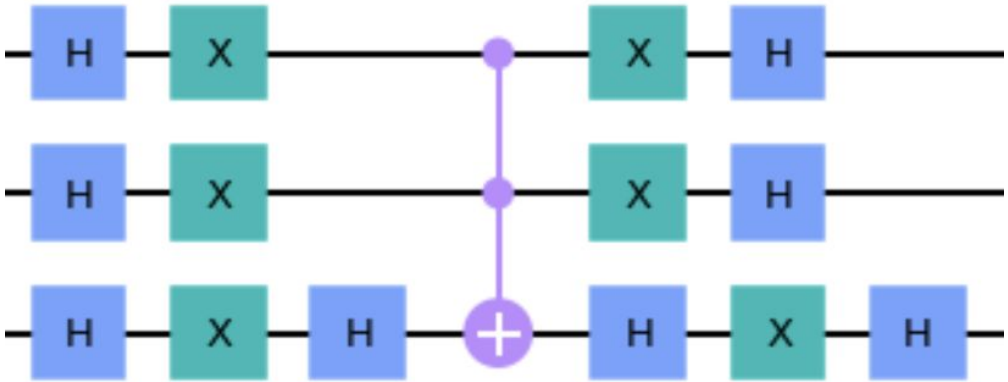


$$U_\omega = I - 2\left|a\right\rangle\left\langle a\right|$$

Standard Amplification Stage

- Use of Hadamard and MCT gates

$$U_s = |s\rangle \langle s| - I$$



Standard Amplification Stage

From our earlier definition of the amplification stage, the unitary operator $U_s = |s\rangle\langle s| - I$. This can be redefined as

$$2(H^{\otimes n}|0\rangle)(\langle 0|H^{\otimes n}) - H^{\otimes n}H^{\otimes n},$$

where $H^{\otimes n}$ is the Hadamard gate being applied to all qubits. Factoring this expression yields:

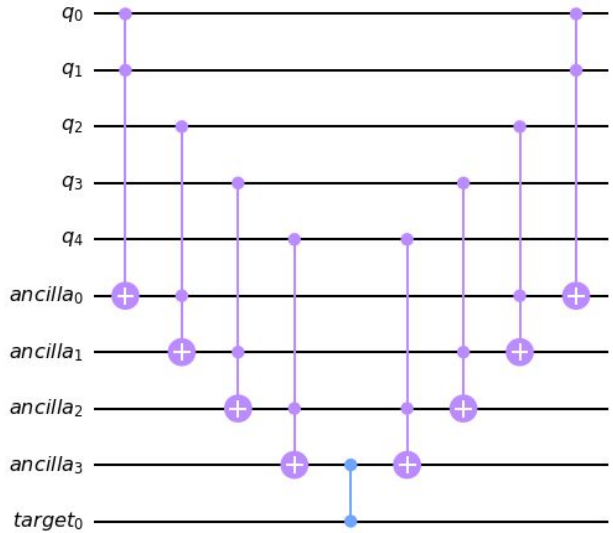
$$H^{\otimes n}(2|0\rangle\langle 0| - I)H^{\otimes n}.$$

Note that U_s and $-U_s$ are equally effective since both unitary matrices rotate the initial state towards a state orthogonal to $|a_\perp\rangle$.

$$-H^{\otimes n}(2|0\rangle\langle 0| - I)H^{\otimes n} = H^{\otimes n}(I - 2|0\rangle\langle 0|)H^{\otimes n}$$

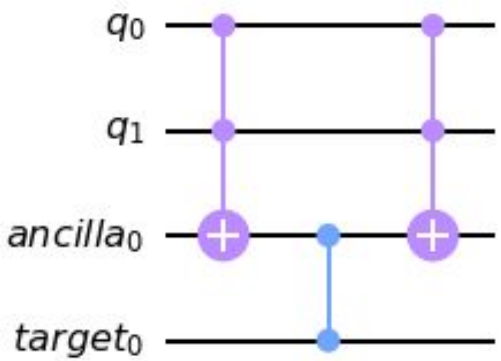
Modified Amplification Stage

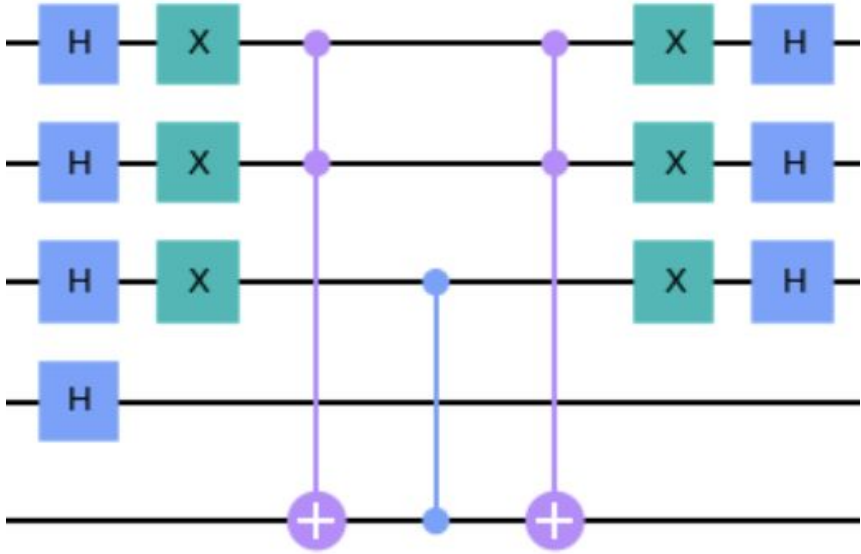
- “compute-copy-uncompute” method
- Replaced MCZ gate



Modified Amplification Stage

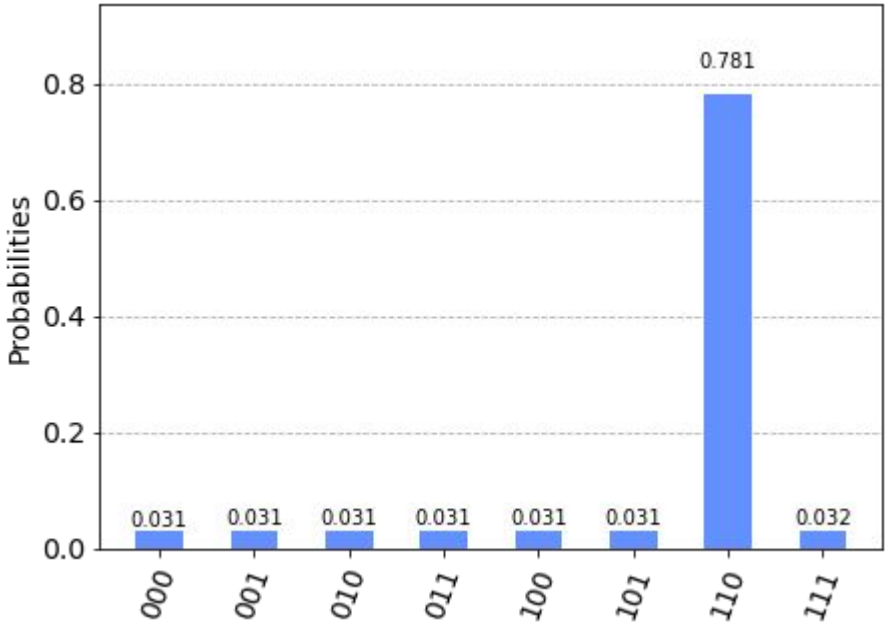
- Applied earlier example

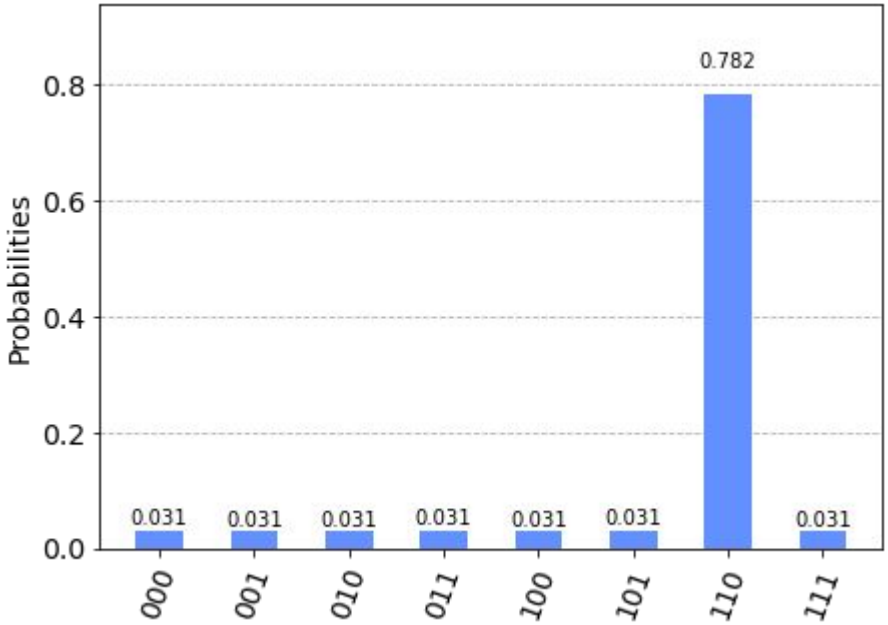




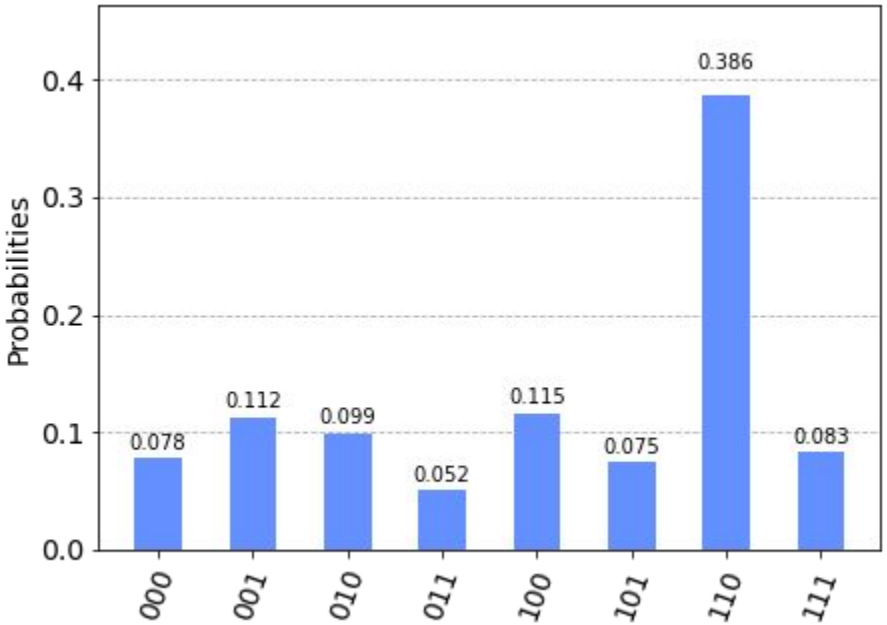
Results Section

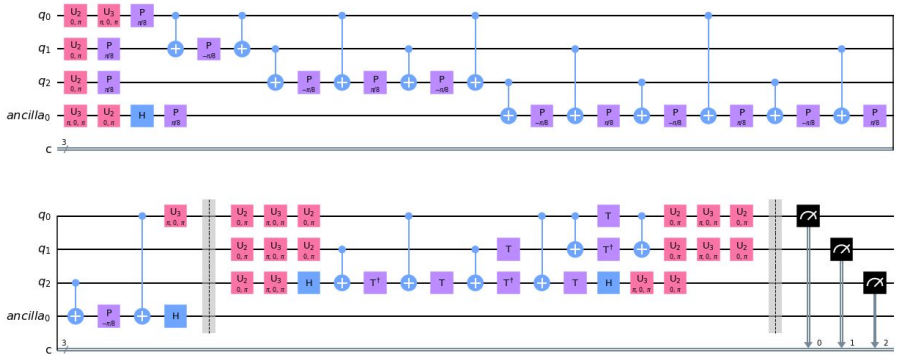
Simulated Results



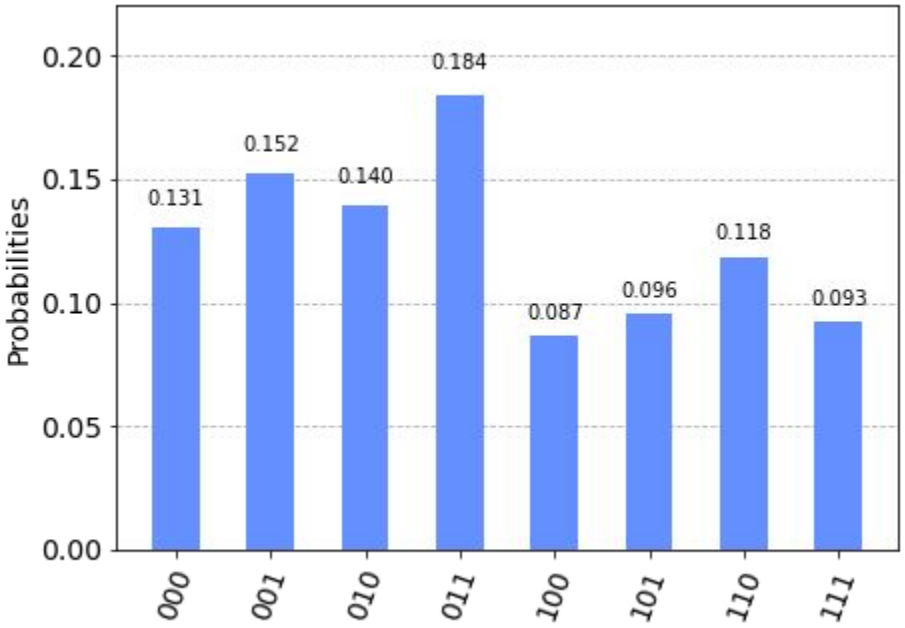


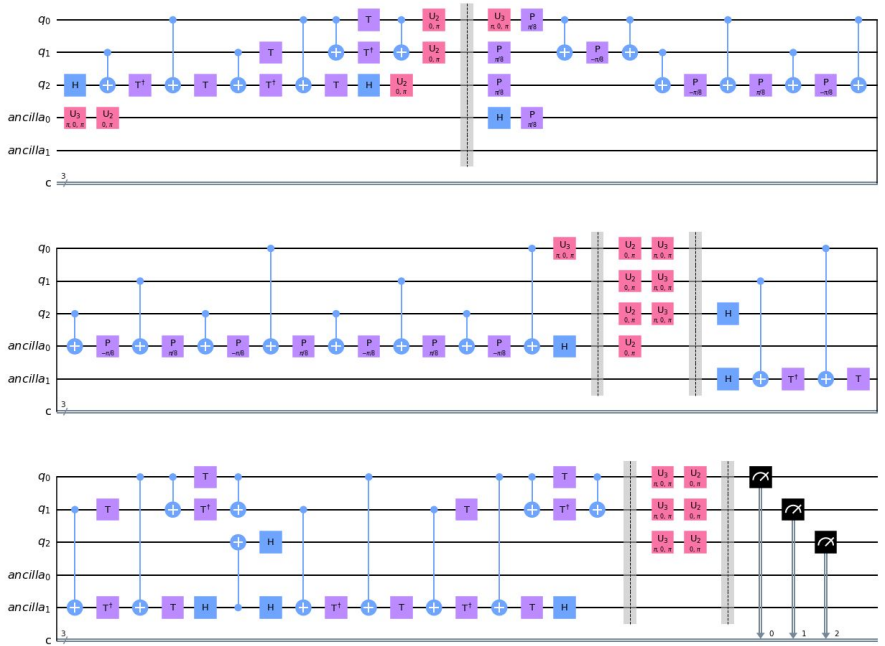
Standard Hardware Results



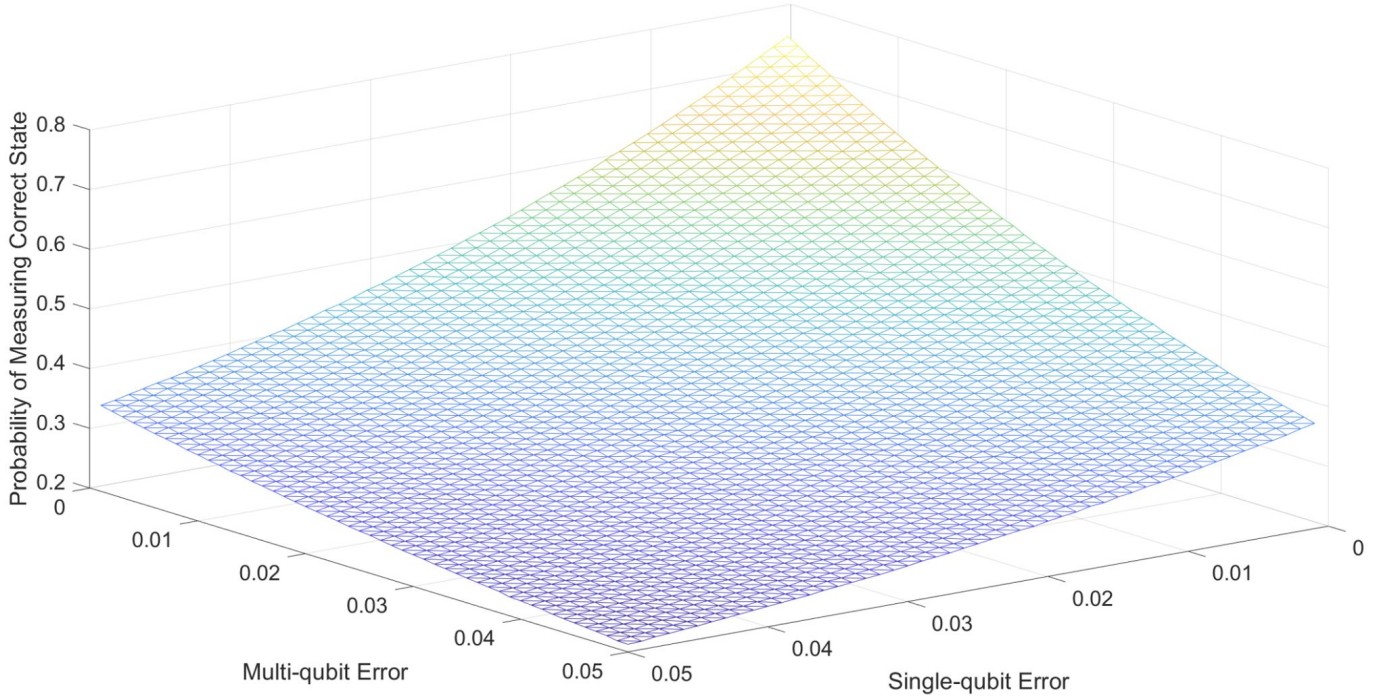


Modified Hardware Results

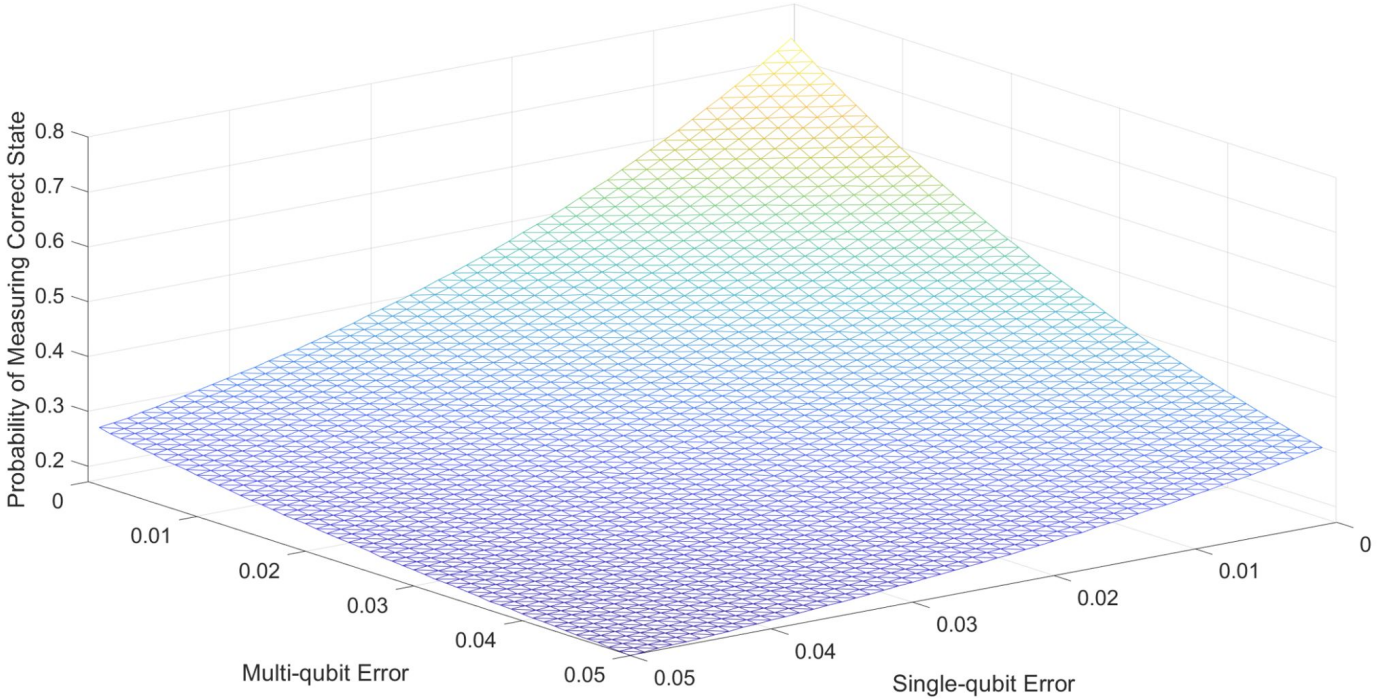




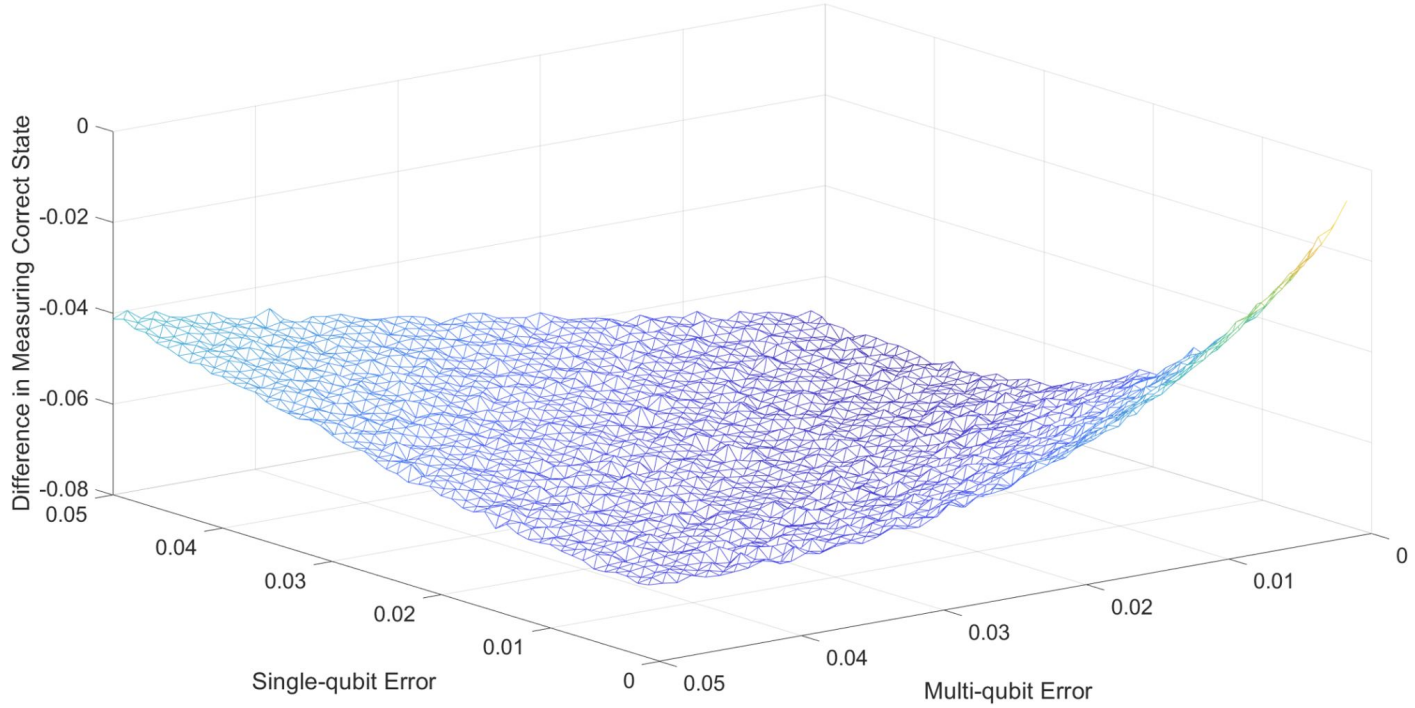
Standard Noisy Results



Modified Noisy Results



Residual Noise



Conclusion

- Standard implementation of Grover's outperforms "compute-copy-uncompute" method
  - May vary for higher-qubit systems
    - Future tests necessary
- High error rates continue to impede hardware results
- Simulated results mirror predicted results